Random Intercept and Random Slope Models

Recap of Important Points

Random Intercept Model

Assumes:
- School effects (intercepts) are random.
- A common linear effect (slope) of the intake variable for all schools.

This implies that school lines are parallel and the intercepts are ‘shrunk’ towards the overall average line.

We have $\sigma^2_{u0} = 0.092$ which is the variance of the intercepts of the 65 schools. Here fitting separate parallel school lines explains $0.092/(0.092+0.566) = 14\%$ of the variation.
Random Slopes Model

Assumes:

- School effects (intercepts) are random.
- The effects of the intake variable (slopes) are random and differ between schools.

This implies that both the intercepts and slopes for each school are ‘shrunk’ towards the overall average line.

Here have $\sigma^2_{u0} = 0.090$ (variance of school intercepts) and $\sigma^2_{u1} = 0.015$ (variance of school slopes). The covariance $\sigma_{01} = 0.015$ and is positive implying higher intercepts $\leftrightarrow$ higher slopes.

The residual (level 1) variance $\sigma^2_e$ has been reduced from 0.566 to 0.554 by introducing random slopes.
What do the parameter values mean graphically?

Here the blue (solid) line is school 53 and the red (solid) line is the (estimated) overall average line $\beta_0 + \beta_1 x_{ij}$.

We obtain the fixed effect estimates $\beta_0 = -0.012$ and $\beta_1 = 0.557$, and the residual estimates $u_{0,53} = 0.646$ and $u_{1,53} = 0.348$.

The red dashed lines are $y = \beta_0 = -0.012$ and $y = \beta_0 + \beta_1 = 0.545$ which correspond to the $y$ values of the red line when $x = 0$ and $1$ respectively.

Similarly the blue dashed lines are $y = \beta_0 + u_{0,53} = 0.634$ and $y = (\beta_0 + u_{0,53}) + (\beta_1 + u_{1,53}) = 1.539$ which correspond to the $y$ values of the school line at $x = 0$ and $1$ respectively.

Note the green line is a linear regression simply including the 70 pupils in school 53 and so the blue line has been ‘shrunk’ from this green line towards the red line.