Laboratory notes

Torsional Vibration Absorber

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Abbreviations

DOF     Degree-of-Freedom
TVA     Torsional Vibration Absorber
NF      Natural Frequency
1 Objectives

This laboratory exercise covers these topics:

- **Vibration measurements** – data acquisition, vibration measurement and excitation.
- **Inertia** – moment of inertia measurement.
- **Stiffness** – stiffness constant, stiffness type (linear nonlinear) determination.
- **Damping** – damping ratio and logarithmic decrement measurement.
- **Free vibration** – 1 DOF torsional system.
- **Forced vibration** – 2 DOF torsional system including TVA,
- **Natural frequencies** – NF for varying conditions of 1 and 2 DOF system.
- **Torsional Vibration Absorber** – theory and experiments demonstrating TVA performance.

2 Apparatus

Most of the vibration tests and measurements will be conducted on the test station shown in Figure 1. The following equipment will be used in the experiments:

- **Data acquisition** – desktop computer, dSpace system and its real time programming environment with application allowing test control,
- **Vibration measurement** – single axis Brul & Kjaer piezoelectric accelerometer,
- **Vibration excitation** – electromagnetic shaker LDS,
- **General equipment** – harmonic signal generator and signal amplifier for the shaker.

Figure 1 TVA test apparatus.
3 Theory

3.1 Background information

This section of the lab notes provides derivation of the basic equations that are used to introduce the theory of TVA. A helicopter blade section is combined with linear spring A, shown in Figure 2a. This structural arrangement can be modelled as 1 DOF system with its DOF represented by the angular coordinate $\theta(t)$. Under normal operational conditions a wing or blade can torsionally vibrate. This effect is induced by electromagnetic shaker with predefined excitation signal. TVA can be used to reduce vibration levels of torsionally vibrating wing. Application of TVA changes original 1 DOF system to 2 DOF system, where second degree of freedom is represented by the coordinate $x(t)$.

![Figure 2 Experimental configuration: a) detail of the test rig (wing section with linear spring and TVA), b) equivalent mechanical representation of the test rig.](image)

3.2 Undamped tuned vibration absorber

The equations of motion for the system shown in Figure 2b) can be constructed using Newton’s 2nd law. In this case, free body diagrams are plotted for both wing section and TVA and these are used to write equilibrium conditions. Both springs are assumed to provide linear forces proportional to $\theta(t)$ and $x(t)$.

![Figure 3 Free body diagrams.](image)
The equation of motion for the wing section is based on moment equilibrium relatively to the wing section hinge “$h$”

$$J \ddot{\theta} + K \theta - a k (x - a \theta) - b F(t) = 0.$$  \hspace{1cm} (1)

where $J$ \([kg.m^2]\) is the moment of inertia of the wing, $K$ \([N.m/rad]\) is the effective torsional stiffness representing wing compliance, $k$ \([N/m]\) is the stiffness of the spring used in the TVA, $a$ and $b$ \([m]\) are geometric offset parameters according to Figure 2b), $F(t)$ \([N]\) is the force provided by the shaker, $\theta$ \([rad]\) and $\ddot{\theta}$ \([rad/s^2]\) is the angular coordinate and acceleration, respectively, of the wing.

The equation of motion of the added mass is based on the general dynamic force equilibrium in vertical direction

$$m \ddot{x} + k x - k a \theta = 0.$$  \hspace{1cm} (2)

where $m$ \([kg]\) is the mass of the TVA, $x$ \([m]\) and $\ddot{x}$ \([m/s^2]\) is the displacement and acceleration, respectively, of the added TVA mass.

Further, a simple harmonic excitation case is assumed

$$F(t) = F_0 \sin(\omega t)$$  \hspace{1cm} (3)

where $F_0$ \([N]\) is the amplitude and $\omega$ \([rad/s]\) is the angular frequency of the harmonic excitation force.

This case of excitation leads to the steady-state response of the wing-mass system in the following form

$$\theta(t) = A \sin(\omega t), \hspace{0.2cm} x(t) = B \sin(\omega t)$$  \hspace{1cm} (4)

where $A$ \([rad]\) and $B$ \([m]\) are the amplitudes of the wing and TVA, respectively.

As there is no damping assumed in above case, possible phase difference between the excitation $F(t)$ and the responses $\theta(t)$ and $x(t)$ will be either $0^\circ$ or $180^\circ$. These differences can be accommodated by positive or negative amplitudes $A$ and $B$, respectively.

Substituting (3) and (4) into equations (1) and (2) gives a system of two linear equations with two unknown amplitudes $A$ and $B$

$$
\begin{align*}
\left( (K + a^2 k) - J \omega^2 \right) A - a k B &= b F_0, \\
(k - m \omega^2) B - k a A &= 0.
\end{align*}
$$  \hspace{1cm} (5)

The system of equations (5) can be written in matrix format

$$
\begin{bmatrix}
(K + a^2 k) - J \omega^2 & -a k \\
-ak & k - m \omega^2
\end{bmatrix}
\begin{bmatrix}
A \\
B
\end{bmatrix} =
\begin{bmatrix}
b F_0 \\
0
\end{bmatrix}.
$$  \hspace{1cm} (6)

Using standard solution techniques for systems of linear equations (e.g. Gauss elimination) provides following solution
The ability to zero vibration amplitude $A$ in (7) for given frequency $\omega=\omega_p$ represents a useful mean of vibration reduction of steady-state torsional vibrations of the wing section. This is particularly important in the case when the excitation frequency $\omega$ is close or identical to the natural frequency $\omega_{0,\text{wing}} = \sqrt{K/J}$ of the original wing without TVA, i.e. resonance of the primary unmodified system $\omega_p \approx \omega_{0,\text{wing}}$. The amplitude $A$ can be zeroed in this undamped case by setting nominator to zero, i.e. $k_m - \omega^2 = 0$. Therefore, to get zero wing response, the tuning condition for TVA parameters is $k = m \omega^2$ or $k/m = \omega^2$, and for the specific case of harmonic excitation the condition is $k/m = \omega_p^2$. However, the concept of TVA is particularly effective in the case of resonant excitation $\omega_p \approx \omega_{0,\text{wing}}$ and in this case the TVA tuning condition can be written $k/m = \omega_{0,\text{wing}}^2$. This means that, for the case of resonant excitation, the undamped natural frequency of the TVA alone should be equal to $\omega_{0,\text{wing}}$. Only this case will be considered in this laboratory exercise.

The working principle of the undamped TVA, for the case of resonant harmonic excitation, is detuning of the original system’s NF from its proximity to the excitation frequency by creating two new NFs located on both sides of the original NF. This situation is illustrated in Figure 4. Inclusion of the TVA in the primary system increases number of DOFs by one. This implies that the resulting wing system will have two DOFs and two NFs.

$$A = \frac{(k - m \omega^2)b}{(K + a^2k - J\omega^2)(k - m \omega^2) - a^2k^2}F_0,$$

$$B = \frac{abk}{(K + a^2k - J\omega^2)(k - m \omega^2) - a^2k^2}F_0.$$  

(7)

Resonance occurs when $A = \infty$, i.e. when the denominator of equation (7) is zero

$$(K + a^2k - J\omega^2)(k - m \omega^2) - a^2k^2 = 0.$$  

(8)
Applying the substitution \( \nu = \omega^2 \), the quartic equation (8) can be converted to a quadratic equation in \( \nu \) and the solution is given by

\[
\nu_{1,2} = \frac{1}{2} \left( \frac{k}{mJ} + \frac{K}{J} + \frac{a^2k}{J} \right) \pm \frac{1}{2} \sqrt{\left( \frac{k}{mJ} + \frac{K}{J} + \frac{a^2k}{J} \right)^2 - 4 \frac{Kk}{Jm}}
\]

(9)

Solving this equation will give two solutions \( \nu_1 \) and \( \nu_2 \). The two undamped natural frequencies of the modified system are \( \Omega_1 = \sqrt{\nu_1} \) and \( \Omega_2 = \sqrt{\nu_2} \).

### 3.3 Logarithmic decrement

Logarithmic decrement is a convenient way of finding the damping of a system. Assuming linear viscous model of damping, it is defined as the natural logarithm of the ratio of two successive peaks of the displacement waveform.

Free vibration of the 1 DOF system decays due to the effect of damping. The frequency of the damped oscillation will be less than the natural frequency of the undamped oscillation. The damped frequency is given by

\[
\omega_D = \omega_0 \sqrt{1 - \zeta^2}
\]

(10)

where \( \zeta \) is the damping ratio (non-dimensional quantity), \( \omega_D \ [rad/s] \) is the damped angular natural frequency and \( \omega_0 \ [rad/s] \) is the undamped angular natural frequency.

**Note:** Typical range of \( \zeta \) in metallic mechanical systems without specific damping treatments is between 0.1 % and 1.0 %, which leads the damped natural frequencies where \( \omega_D \approx \omega_0 \). This is a practical implication of so called small damping assumption.

The effect of free decaying 1DOF vibration is shown in Figure 5. In the case of viscously damped system, this response can be mathematically described by an exponentially decaying sinusoid

\[
y = Y e^{-\zeta\omega_0 t} \sin(\omega_0 t + \phi)
\]

(11)

where \( Y \ [m] \) is the amplitude and \( \phi \ [rad] \) is the phase angle.
It is useful to study the change in vibration amplitudes in the two successive peaks of the free response shown in Figure 5. The two arbitrary successive amplitudes are as follows

\[ Y_i = Y e^{-\zeta \omega_0 t_i}, \quad Y_{i+1} = Y e^{-\zeta \omega_0 (t_i + \tau_d)} \]  

(12)

where \( \tau_d \) [s] is the period of free damped oscillation.

Natural logarithm of the ratio of these two successive peaks is called the logarithmic decrement \( \Lambda \)

\[ \Lambda = \ln \left( \frac{Y e^{-\zeta \omega_0 t_i}}{Y e^{-\zeta \omega_0 (t_i + \tau_d)}} \right) = \ln(e^{-\zeta \omega_0 \tau_d}) = \zeta \omega_0 \tau_d. \]  

(13)

Further, considering basic relationship between the period and angular frequency, \( \omega_d \tau_d = 2\pi \), following relationship can be derived for logarithmic decrement

\[ \tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_0 \sqrt{(1 - \zeta^2)}} \]  

(14)

\[ \zeta \omega_0 \tau_d = \zeta \omega_0 \frac{2\pi}{\omega_0 \sqrt{(1 - \zeta^2)}} \]

and

\[ \Lambda = \frac{2\pi \zeta}{\sqrt{(1 - \zeta^2)}} \approx 2\pi \zeta \]  

(15)

However, due to clarity of measurement, it is usually useful to consider differences between response peaks (of the same sign) separated more than one period of free damped oscillation, i.e. \( N\tau_d \), where \( N > 1 \) is the positive integer. It can be shown that the logarithmic decrement in this case will be

\[ \Lambda = \frac{1}{N} \ln \left( \frac{Y}{Y_{i+N}} \right). \]  

(16)

Figure 6 shows generalisation of this concept, where the whole time response signal \( y(t) \) is converted to logarithmic domain, i.e. \( \ln(|y(t)|) \). The implication of the above theory is that the envelope of this signal will be line instead of exponential curve. The slope of this curve will be indicative of the damping in the system.
3.4 Calculation of the moment of inertia $J$

Considering the wing system in its original state, i.e. no TVA, and without external excitation, i.e. $F=0$, its equation of motion adopts the form

$$J \ddot{\theta} + K \theta = 0. \quad (17)$$

where $J$ [kg m$^2$] is the moment of inertia of the wing, $K$ [N m/rad] is the effective torsional stiffness and the undamped natural frequency of this system is $\omega_{\text{0,wing}} = \sqrt{K/J}$.

Assuming knowledge of $K$, e.g. via static load-displacement test, and $\omega_{\text{0,wing}}$, e.g. via free vibration test, it is possible to determine experimentally the value of $J$ and use it in the previous calculations as required.

**Note:** The *moment of inertia* is a dynamic parameter with units [kg m$^2$]. It characterises object’s resistance to the changes in its rotational state with respect to a particular axis of rotation. It should not be confused with a quantity named *polar moment of inertia*, which is a static parameter with units [m$^4$] and is used to express resistance of shafts or bars with circular cross-section to torsional deformation. Polar moment of inertia is related to another static quantity called *area moment of inertia* with units [m$^4$].

4 Test procedure

4.1 Measurement of logarithmic decrement

i. Attach accelerometer to wing (if not already attached), behind the axis of rotation.

ii. Displace wing, and allow motion to decay (i.e. apply non-zero initial conditions).

Record motion on the computer as you release the wing.

iii. Measure logarithmic decrement $\Lambda$.

Calculate $J$ from natural frequency.

Note that damping will be small, so measure over several cycles.

**Note:** Logarithmic decrement is a measure of decaying displacement. Therefore, strictly speaking, it should be calculated from the record of displacement decay. Clearly, accelerometers measure decaying acceleration of the wing. However, because the motion is a lightly damped sinusoid, double integration of the acceleration to give the displacement will provide a record which very similar to the original acceleration record. Therefore, it is an acceptable approximation to use accelerometer data to measure the logarithmic decrement. An example of relatively extreme situation with 25% damping ratio (in practical applications usually less or around 1%) is demonstrated in Figure 7. Despite large
amount of damping in the system, the envelopes of both the displacement and acceleration, which are indicative of damping, have very similar slopes.

![Comparison between normalised displacement, velocity and acceleration for system with 25% damping ratio.](image)

**Figure 7** Comparison between normalised displacement, velocity and acceleration for system with 25% damping ratio.

### 4.2 Vibration absorption

i. While using harmonic excitation waveform, increase frequency of excitation until it approaches the natural frequency.

ii. Once the natural frequency is reached for the reference wing system (1 DOF system), add mass-spring absorber until vibration is completely absorbed. Weigh mass + scale-pan + one third of mass of spring.

iii. Change the frequencies of the shaker until two new resonances of the combined wing + absorber system are found (2 DOF system).

iv. Record all the measured frequencies.

v. Repeat this vibration absorption exercise with elastic bands replacing the springs.

vi. Measure the stiffness of the elastic bands, either using Hook’s law \( F = kx \) or the measure natural frequency of the isolated absorber subsystem and use \( \omega_{\text{absorber}} = \sqrt{\frac{k}{m}} \).

### 5 Report formatting

You should hand in a technical note describing the above laboratory exercise. This should be typed using 12 point Times font with 1.5 line spacing. Marks will be given for presentation and technical content. A suitable format for the report is:

- **Executive summary** – a brief (under 1 page) summary of the lab and the key findings.
- **Introduction** – this should be very brief (well under 1 page) and to the point.
- **Results** – you should select and present the key results in a suitable and succinct format (2 pages maximum).
• Discussion and Conclusions – discuss the key findings and address the points (i)-(v) below (2 pages maximum). Note that this is where most of the technical marks are.

Point to observe and evaluate:

i. Solve equation (9) and compare the results with the experimental values.

ii. Is the experimentally determined mass for zero response close to $m = k/\alpha^2$?

   If the results are different, discuss the reasons for the discrepancy.

iii. Discuss the performance of the undamped TVA.

iv. Discuss the action of damping on the system and how it might affect the results.

v. Discuss the non-linear aspects of the experiment.

6 Test data

Previously determined experimental and physical parameters:

• Red test rig: wing spring stiffness $K_{red} = 119.2$ [Nm/rad],

• Blue test rig: wing spring stiffness $K_{blue} = 82.1$ [Nm/rad],

• Distance of the TVA from hinge $a = 0.56$ [m].